DETERMINATION OF WORK-IN-PROGRESS INVENTORY COST IN A MULTI-STAGE, MULTI-PRODUCTS MANUFACTURING PROCESS

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ABSTRACT

This paper examined the Multi-stage, multi-products manufacture faces problems of great uncertainty in work-in-progress inventory control. Realizing the significance of such uncertainty, work-in-progress inventory of Spring Diamond Tools Limited, Lagos, that produces three products: Bearing (P_1), Bolt (P_2), and Gear (P_3) in multi-stages were studied for a period of six (6) months. Attempts were technically made to analyze and determine its WIP cost (IW) within the period of six (6) months. Results reviewed that the WIP cost (IW) for an average of one month was N26.72m at the total production cost (Tc) of N33.42m. Evaluation of the ratio of the WIP cost (IW) to the total production cost (Tc) yielded 1: 1.33 which indicate that WIP accounts for huge sum of the overall production cost of the company. To enhance WIP inventory determination and its control policies in a multi-stage, multi-product manufacturing outfit, a mathematical model is proposed in this thesis, and attempts were made to use it in solving the WIP problem of the outfit using Tore software. Results reviewed a WIP cost (IW) of N23.85m as optional solution as compared to N26.72m derived.

Keywords: Inventory, workstation, multi-stage, work-in-progress

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1. INTRODUCTION

A manufacturing process consists of work stations that carry out specific operations to create a predesigned product. There are many types of manufacturing process that have been studied from different points of view. In these processes, a product is produced in a sequence of manufacturing operations separated by inventories of work-in-process (WIP). After completing an operation at a stage, the processed items are transported to the work-in-process inventory for the subsequent production stages.

The customer demand is met up from the final production stage. It is a special case of an assembly network in which an item requires processing in strictly ordered stages. There is a tendency in multi-stage production systems for the work-in-process to increase at all stages unless the production flow is synchronized (Sarker, 2002).

These processes are classified into a job shop, flow-line, continuous production processes etc. A job shop manufacturing process is a facility that produces a wide range of products. These products require different processing sequence.

Optimal production management aims to minimize work-in-process. Work-in-process requires storage space, represents bound capital not disposable for investment and carries an inherent risk of earlier expiration of shelf life of the products (Forgety, 2001). A queue leading to a production step shows that the step is well buffered for shortage in supplies from preceding steps, but may also indicate insufficient capital to process the output from these preceding steps.
However, in a job shop manufacturing process, the products require processing in a well-defined sequence on a set of machine(s) work station(s). In these processes scheduling and routing represent an enormous problem facing production control (Shebrooke, 2004). On the other hand, a flow line manufacturing process is a facility that produces a few discrete products. All products in this process are required to go through all machines in the same sequence which allows the production of large quantities.

Finally, a continuous process manufacturing process utilizes highly specialized equipment to produce products such as petroleum, petrochemicals etc. Nevertheless, each manufacturing process has its own characteristics and requirements in terms of production and inventory control. Overall, the objective of a production system is to transform raw materials or subcomponents into a final product which is delivered to customers.

Therefore, the production process of a product consists of several stages such as raw materials inventory, machining (processing), and buffer inventory (WIP) and final product inventory. These stages of a production process are as shown in fig 1.1.

Inventory, which is the stock of any item or resource used in an organization, is an essential component of a production process environment.

It exists in these systems in three forms as follows: raw material, work-in-process (WIP), spare parts/supplies and finished goods inventories. Raw material inventory includes items that require some types of processing to manufacture a final product. Raw material inventory is the amount of raw materials that are in the possession of the owner during a specific period (Anderson, 2001).

Some final products of production processes are considered as raw material or subcomponents to other production processes.

Work-in-process (WIP) inventory exists in production processes where different types of raw materials are processed into finished products. Also, it exists as buffer between two work stations if the output of a work station is transferred in batches to the next work station(s).

Finished product inventory represents items that are held at the manufacturing storage facility waiting to be shipped to the customers. All manufacturing and marketing companies hold finished goods inventories in various locations and all through F G supply chain (Liu and Yao, 2004) which finished goods move through the supply chain from the point of manufacturing until it reaches the end customer, depending upon the sales and delivery model, the inventories may be owned and held by the company or by intermediaries associated with the sales channels such as traders, trading partners, stockists, distributors and dealers etc. Hence, inventories are used to balance variability and uncertainty in supply and demand. Notably, as the size of inventory increases, the cost of managing and holding inventory increase as well.

The costs that associate with holding inventory are as follows: storage space, handling, insurance, taxes etc. Therefore, production system aim at reducing the size of inventory in all three forms of inventory to minimize the total cost of production. The purposes of inventory be it raw material, work in process (WIP) or finished product are as follows:

1. To maintain independence of operations: A supply of materials at a work centre allows that centre flexibility in operations for example, because there are costs for making each new production set up this inventory allows management to reduce the number of set-ups. Work station on an assembly line usually is not in dependent between raw material and products to work on are fed at the line speed. There may be none or only a few extra products to work on in the event the worker performs either faster or slower than line speed or if the workstation upstream slow down output. The unit completed at a workstation passes to the next person.
II. To meet variation in product demand. If the demand for the product is known precisely, it may be possible (though not necessarily economical) to produce the product to exactly meet the demand. Usually, however, demand is not completely known and a safety or buffer stock must be maintained to absorb variation (Anderson, 2001).

III. To allow flexibility in production scheduling. A stock of inventory relieves the pressure on the production process to get the goods out. This causes longer lead times which permit production. High set-up costs for example, favour the production of a larger number of units once the set-up has been made (Tersine 2007).

IV. To provide a set safeguard for variation in raw materials delivery time when material is ordered from a vendor, delays can occur for a variety of reason; a normal variation in shipping time, a shortage of materials at the vendor’s plant causing backlogs an unexpected strike at the vendor’s plant or at one of the shipping companies, a lost order, or a shipment of incorrect or defective materials (Heuts and Munk, 2002).

V. To take advantage of economic purchase order size. Obviously, there are costs to place an order labour, phone cell, typing, postage/faxing etc. Therefore, the larger the size of each order, the fewer the number of order that need be written. Also, the non-linearity of shipping cost favour larger orders, the larger the shipment, the lower the per-unit cost. (Tersine 2007) Assume a manufacturing process where materials are routed through a series of steps, value is added at each step and time is consumed in between steps such as schedule, lead time, queue time, quality control time etc. Also assumed that the product is manufactured in discrete, equal-sized batches, work-in-process for a single batch can be graphically depicted as shown in fig 2.

![Diagram of work-in-process inventory](image)

**Fig.2** Wip for single stage (Sipper and Shapira, 2002.)

In this research, a work-in-process inventory in a production system that consists of four work stations and three products is studied. The work-in-process inventory of multi-product multi-stage manufacturing system typically studied at Spring Diamond Tools Limited, Lagos State for a period of six months and attempts were made to determine the optimal cost associated with inventory of the organization under the time in review.

2. WORK-IN-PROCESS INVENTORY CONTROL MODELS
Sarker (2002) considered a manufacturing system which procured raw materials and converted them to finished products of varying demand. The author has proposed a decision rule to determine the production start time, lot and batch sizes with minimum cost of ordering raw materials, manufacturing set-up, raw material and finished product holding.

Kim and Kiy (2001), synchronized the production flow in a serial production process by transferring a lot from a stage to the next with equal-sized batches. The batches are transported from a stage to the next processing without waiting for the entire production lot to be processed at the earlier stage before being moved to the next stage. They developed a model as thus,

\[ WIP = \sum_{i=1}^{n} \frac{Q_i P_2}{P_i} \]

Their model for determining average work-in-progress WIP is stated thus:

\[ WIP_{ave} = \frac{1}{n} \left\{ \sum_{i=1}^{n} Q_i + \frac{P_2}{P_1} \sum_{i=1}^{n-1} Q_i \right\} \]

Where

- \( n \) = no of batches,
- \( Q \) = batch size
- \( P_1 \) = processing time of machine1
- \( P_2 \) = processing time of machine2

This is mainly used for a two stage manufacture which is dependent upon the batch number and the applicable conditions of processing times.

Notably, this model is restricted to a two-stage manufacturing process.

Assuming a fixed set up cost for all stages and a fixed cost of transporting a batch through all stages, we developed an economic production lot size model taking into account the set-up and transportation times. Gunesekaran and Goyal, 1991, developed a model of processing a single product in any number of serial production stages. This allows the combination of equal and unequal batch size is the largest and does not exceed the transport equipment capacity. A heuristic procedure based on the concept of differentiation of the cost function was developed to determine the economic lot size and batch sizes for each stage. But this heuristic provided no way of estimating how close it was to the actual minimum cost solution. It merely gave comparison with earlier published heuristics. Furthermore, the procedure did not naturally deal with certain specific cases that could easily occur, for instance, zero transportation times for batch transfer between stages and equal production rates at successive stages.

Considering these specific cases, Zhang and Gerchack, (2004) had developed a modification to the model of Goyal, which enables a number of properties that the optimal solution must satisfy to be determined. An algorithm giving the optimal solution was then derived based on these properties.

Thus the model of Zhang and Gerchack, (2004) is a particular case of the modified model of the Kim and Kiy (2001), except in the consideration of a set up and transportation times and avoidance of the capacity constraint on the transport equipment. To cope with model of Kim and Kiy (2001), Zhang and Gerchack, (2004) extends the model to include set up and transportation times.

### 3. MULTI STAGES, MULTI PRODUCTS MANUFACTURING PROCESS

Multi-stage production is common in manufacturing industry. The fundamental challenge of multi-stage production is the propagation and accumulation of uncertainties, which influences the conformity of the output (Portues, 2000).

Uncertainty is always present in manufacturing environment. These uncertainties affect the performance of a process, including its service level in terms of fill rate or delivery lead time, which in turn affects the bottom line of an enterprise in today’s competitive world environment (Liu and Yao, 2004).

An item is said to undergo multi-stages when during the processing of the item, it passes through different workstations in which case, value is added at each stage of production. A workstation may comprise of one or more machines involved in product processing. In some cases, multi-stage production might involve the creation of one item or product. This is known as multi-stage, single product manufacturing process.
Determination of work-in-progress inventory cost in a multi-stage, multi-products manufacturing process

But in multi-stage, multi-products the items through many workstations of different number of machines separated by the work-in-progress inventory. Here, the items require processing in strictly ordered stages. Sarkar (2000), maintained that there is a tendency in multi-stage production process for the WIP to increase at all stages unless the production flow is synchronized.

4. PROBLEM STATEMENT

In recent times, multi-stage, multi-products manufacture has faced problems of great uncertainty in work-in-progress inventory control. Realizing the significance of such uncertainty in the production process, calls for attempts to resolve the problem of work-in-progress inventory in a multi-stage, multi-product manufacture as capital tied up. It is obvious that determining the WIP cost (IW) would enhance productivity inventory control policies, as well as helping the management of such outfits to estimate with ease their WIP cost (IW). These challenges posed by WIP, call for in-depth analysis of its behaviours and determination in this thesis.

5. RESEARCH METHODOLOGY

Steps taken in the research were questionnaire, personal interview and observations in the production lines. Our sample is a survey carried out at Spring Diamond Tools Ltd, Lagos, Nigeria.

A questionnaire was designed to identify views of the production manager about Work-in-Progress (WIP) cost in inventory management (IM).

A total of 300 questionnaires were distributed and 262 retrieved which represents 87.3%.

The data obtained from the survey were analyzed using linear programming approach. Table 1 shows the cost element per product from July – December 2012.

Table 1. Cost Element Per Product

<table>
<thead>
<tr>
<th>Cost Element</th>
<th>Cost Per Product (N/M)</th>
<th>No. of Product</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tc</td>
<td>P1 = 8.94</td>
<td>P2 = 9.14</td>
</tr>
<tr>
<td>Cs</td>
<td>0.44</td>
<td>0.51</td>
</tr>
<tr>
<td>Cim</td>
<td>2.00</td>
<td>2.40</td>
</tr>
<tr>
<td>Quantity Production</td>
<td>300035</td>
<td>325000</td>
</tr>
</tbody>
</table>

6. DETERMINATION OF THE WORK-IN-PROGRESS

In the determination of the work in progress for the multi-stage, multi-products outfits, the following assumptions were made

i. Demand for the products is uniform, deterministic and known.

ii. Set-up cost per set-up is independent of set-up sequences

iii. Once the product goes out of control, the machines automatically stop producing defective products.

iv. There is no finished product inventory cost as the products will be dispatched once the processing is completed at the final stage.

However, the total cost of production consists of the following

Set up cost

Imbalance cost

Cost due to work-in-progress

Set-up cost

The total set-up cost considering all products and stages is given by

\[ \sum_{i=1}^{M} \sum_{j=1}^{N} (D_{ij}A) = Cs \quad \ldots \ldots \quad 4.10 \]

The processing time for a batch is given by

\[ T_i = \Phi_t \times t_{ij} \quad \ldots \ldots \quad 4.12 \]
Suppose the operator is a server when there is \( \delta_j \) machines at stages \( j \), every time a machine drifts, it has to be serviced by the operator from \( \text{m/m/1} \) queuing theory, the total time spent (waiting time per drift) by a batch per drift can be estimated as

\[
\frac{1}{(\beta_{ij} - \alpha_{ij})} \quad \text{.........} \quad 4.13
\]

Average time spent by the batch due to process drift while processing that batch can be estimated

\[
T_{ij} = \left\{ \frac{\sigma_{ij}}{(\beta_{ij} - \alpha_{ij})} \right\} \quad \text{.........} \quad 4.14
\]

The number of production cycles per unit time for each product at a given stage is represented by

\[
R_i = \frac{B_i}{Q_i} \quad \text{.........} \quad 4.15
\]

Since process drift may occur at any time during the processing of a particular product, the value of per unit product at which the drift occurs may be difficult to obtain. Hence, the average cost per unit product has been accounted to compute the imbalances.

\[
C_{ij} = \frac{c_{ij} - \frac{c_{ij} + c_{ij}}{2}}{2} \quad \text{.........} \quad 4.16
\]

**Cost due to imbalance**

The main aim of any production process tends to implement job production is to balance the production rates between successive production stages. The aspect of quality at the source have been modeled by loyal and Gunasekaran (1999).

However, the production rate for a particular product from that stage \( j \) can be estimated.

\[
\lambda_{ij} = \frac{e_{ij} \times s_{ij}}{t_{ij}} \quad \text{.........} \quad 4.17
\]

Where: \( \sum_{j=1}^{N} \sum_{i=1}^{M} \delta_{ij} = 1 \) for \( j = 1, 2, \ldots, N \)

To achieve a balance among production rates, I have considered a penalty cost which encompasses all relevant cost associated with imbalance in production rate.

The cost due to imbalance in production rate between a given stage and the next stage is given

\[
|\lambda_{ij} - \lambda_{ij} + 1| \times \Phi_i \quad \text{.........} \quad 4.18
\]

Where: \( \Phi_i = \text{Penalty cost} \)

Total cost due to imbalance is given by:

\[
\sum_{j=1}^{M} \sum_{i=1}^{N} |\lambda_{ij} - \lambda_{ij} + 1| \times \Phi_i \quad \text{.........} \quad 4.19
\]

Set up cost considering all stages is given by:

\[
\sum_{j=1}^{M} \sum_{i=1}^{N} \left( \frac{B_i}{Q_i} \right) \Lambda = Cs \quad \text{.........} \quad 4.20
\]

Where: \( A = \text{set up cost per set-up for product \( i \) at \( j \) stage.} \)

**Bearings**

Bearing cost due to imbalance considering all stages is given by

\[
C_{\text{bmn}} = \sum_{i=1}^{M} \sum_{j=1}^{N} |\lambda_{ij} - \lambda_{ij} + 1| \times \Phi_i \quad \text{.........} \quad 4.21
\]

But

\[
\lambda_{ij} = \frac{e_{ij} \times s_{ij}}{t_{ij}} \quad \sum_{i=1}^{M} \sum_{j=1}^{N} \delta_{ij} = 1 \quad \text{for} \ j = 1, 2, 3 \ldots, N
\]

\[
L_{ij} = T_{ij} \left\{ \frac{B_i}{(\beta_{ij} - \alpha_{ij})} \right\} \quad \text{.........} \quad 4.22
\]
7. FORMATION OF LINEAR PROGRAMMING FOR WORK IN PROGRESS INVENTORY

The work in progress inventory model development below is for a multistage, multi-products manufacturing process and can be solved using a TORA software

\[ \text{Minimize } Z = \sum_{i=1}^{n} (T_{1i} x_{1} - (C_{e} x_{1}) - (C_{oa} x_{1}) \]

Subject to \( \sum_{j=1}^{N} x_{1} = S_{i} \) for \( i = 1,2, \ldots, n \) ........................ 4.28
And \( x_{1} \leq 0 \) for all \( i = 1,2,3 \ldots,n \)

Solving this WIP cost (IW) problem with linear programming, we use double-subscripted decision variables with:

- \( x_{11} = \text{Number of } P_{1} \text{ with respect to } T_{c} \)
- \( x_{12} = \text{Number of } P_{2} \text{ with respect to } T_{c} \)
- \( x_{13} = \text{Number of } P_{3} \text{ with respect to } T_{c} \)
- \( x_{21} = \text{Number of } P_{1} \text{ with respect to } C_{e} \)
- \( x_{22} = \text{Number of } P_{2} \text{ with respect to } C_{e} \)
- \( x_{23} = \text{Number of } P_{3} \text{ with respect to } C_{e} \)
- \( x_{31} = \text{Number of } P_{1} \text{ with respect to } C_{oa} \)
- \( x_{32} = \text{Number of } P_{2} \text{ with respect to } C_{oa} \)
- \( x_{33} = \text{Number of } P_{3} \text{ with respect to } C_{oa} \)

Where \( Z = \text{Objective function that determines the WIP cost} \)
- \( X = \text{Cost variable} \)
- \( S = \text{Constraint or restriction placed upon the problem} \)

Constraints
Total cost
Set-up cost
Imbalance cost
Number of products

From the survey data table 4.3, the objective function can be represented \( \text{Minimize } Z = (8.84 x_{11} + 9.14 x_{12} + 8.72 x_{13}) - (0.44 x_{21} + 0.51 x_{22} + 0.54 x_{23}) - (2.40 x_{31} + 2.40 x_{32} + 2.40 x_{33}) \)

................................................. 4.29 ie. the WIP cost (IW) of the three products.

Subject to

\[ x_{11} + x_{12} + x_{13} \geq 3 \]
\[ x_{21} + x_{22} + x_{23} \geq 3 \]
\[ x_{31} + x_{32} + x_{33} \geq 3 \]
\[ x_{11} + x_{21} + x_{31} \geq 300035 \]
\[ x_{12} + x_{22} + x_{32} \geq 325000 \]
\[ x_{13} + x_{23} + x_{33} \geq 304085 \]

ie. Cost of the product quantity of the three products \( (P_{1}, P_{2}, P_{3}) \) and \( x_{11}, x_{12} \ldots \ldots x_{13} \) all such values are \( \geq 0 \)

8. RESULTS AND DISCUSSION

From survey data table 4.1, the WIP cost (IW) was calculated to be
IW for \( P_{1} + P_{2} + P_{3} = N\text{26,712,293.00k} \)
And the total production cost = \( N\text{35,414,133.00k} \)

Ratio of the WIP cost (IW) to the total production cost \( Tc (\mu) \)

\[ \mu = 1 : 1.33 \]

From the result, it can be seen that the work-in-progress cost accounts for about 75% of the overall cost of the multi-stage, multi-product manufacturing process.
The WIP cost (IW) accounts for huge sum in manufacturing processes be it multi-stage, single product process; single-stage, single product process, etc.

The wip cost (iw) model results
The WIP model developed in chapter four was solved with TORA software package, which provides the following information:

i. Information about the objective function
   a. Objective function optimal value
   b. Coefficient ranges. The range of optimality for each coefficient provides the range over which the current solution will remain optimal.

It is advisable that managers should focus on these objective coefficients that have a narrow range of optimality and coefficients near the end point of the range.

ii. Information about the decision variables
   a. Their optimal values
   b. Their reduced costs

iii. Information about the constraints
   a. The amount of slack or surplus
   b. The dual prices that represent the improvement in the value of the optimal solution per product increases in the right hand side.
   c. Right-hand ranges (ranges of feasibility) that represent the range over which the dual price is applicable. As the RHS increases, other constraints will become binding and limit the change in the value of the objective function solving the above WIP cost model using TORA software package will result.

IW cost Z = \( \$23,847,315 \)

\[ x_2 = x_{12} = 3.00, x_4 = x_{24} = 280032, x_5 = x_{25} = 224997, x_6 = x_{26} = 284055, x_7 = x_{37} = 3.00 \]

While other variable has zero value (appendix A)

The amount (\( \$23,847,315 \)) represents the minimum monthly work-in-progress cost (IW) for the company to produce the three products. From the computer result sheet, variable \( x_i = x_{11} \) which is number of \( P_i \) with respect to \( T_2 \) is zero, this shows that \( P_i \) with objective coefficient of 9.70 has no objective value contribution.

9. CONCLUSION AND RECOMMENDATIONS

In conclusion therefore, this paper has reviewed that work-in-progress inventory cost is a crucial and important determinant of the manufacturing outfits be it multi-stage, multi-products multi-stage single products, single stage multi-products etc. WIP requires serious attention than other types of inventories such as Raw materials, finished good and supplies/spare parts.

Furthermore, it has been observed that WIP cost (IW) accounts for about 75% of the overall production cost (TC). Appropriate control methods (model) can be employed to analyze and determine the WIP cost (IW). Such models is of great importance to the managements of the manufacturing outfits, in their WIP cost (IW) determinations. TORA software has been used in solving the suggested model in this thesis to arrive at the optimal WIP cost of the production outfit. The sensitivity analysis gives the insight of the process behavior dependent upon certain variables.

However, the WIP cost analysis and the developed model are based on a number of assumptions and approximations. This implies that there are avenues for further investigations to enhance the accuracy of determining the WIP cost (IW) of a multi-stage, multi-products manufacturing processes through the model proposed in this thesis and use of other mathematical approaches. It is important to note that various insights could be derived from the model for different production situations on different related issues like investing in set-up reduction programs, quality control etc. and their implications on the performance of the multi-stage, multi-products manufacturing processes.

Meanwhile, the use of the proposed approaches in this thesis in WIP cost determination of any multi-stage, multi products manufacturing process should be done with caution. Any error arising due to the proposed approaches is not the responsibility of the proposer and his supervisors.
REFERENCE