Determination of the Optimal Order Quantity for Multi-Item Inventory

A.C. Uzoch

Department of Mechanical Engineering, Federal University of Technology Owerri PMB 1526 Owerri, Imo State, Nigeria.

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Abstract

This paper attempts to find the optimal order quantity for a multi-item inventory system. Two cases were considered: The determination of the optimal ordering quantities without constraints and determination of the optimal order quantities (where there are limited funds for the purchase of eighteen (18) raw materials, thus creating a budget constraint. Economic Order Quantity (EOQ) model is developed and the Lagrangean multiplier is used to bring the order quantity within the acceptable limits of the constraint. The result is compared with the existing inventory policy of the company. The total annual inventory costs incurred currently by the company is N35,097,340.83, and the total annual inventory costs calculated using the developed EOQ model is N13,850,836.78, thus reducing the total inventory costs by N21,246,504.05, which is a savings of 60.54%. In the case where there is a budget constraint the Economic Order Quantity (EOQ) model was developed and the Lagrangean multiplier method is used to determine the EOQ. The total procurement cost obtained was N2,322,386.00, which satisfied the constraint.

Keywords: Multi-item Inventory, constraints, EOQ, Optimal Cost.

1.0 Introduction

Production industries and manufacturing firms in today’s market face the dual challenge of cutting costs as well as being responsive to market demands. As industrial competition, customer expectations and the need for good supplier relationship increases, managers must continually search for ways to create and keep customers and at the same time increase the incoming revenue of their companies (Chen and Chen, 2005). In a nutshell, companies willing to stay in business must be as dynamic as the markets in which they are in. This has driven manufacturers of goods and services to continually search for ways to produce goods or provide services at acceptable quality standards while continually searching for ways to reduce in-process costs.

A major source of these in-process costs are as a direct result of inventory costs. Thus there is a growing concern with regards to the need for the proper management of the resources of an industry, while meeting required demand levels and at the same time reducing the costs associated with inventory, (Teng, 2002).

Inventory generally refers to materials in stock; a detailed list of movable goods such as raw materials, materials in process, finished products, general supplies and equipment, etc (Verma, 2002). The necessity of inventory control in the effective management of inventory is to maintain a reserve of goods that will ensure that manufacturing continues smoothly, according to the production plan, based on sales requirements and at the lowest possible costs incurred. Ineffective and improper management of inventory results in loss. Losses from improper inventory control include purchases in excess of what is needed, the cost of slowed down production resulting from material not available when wanted (Verma, 2002).

1.1 Inventory and the EOQ Model

Telsang (1998) defines inventory as ‘any material in stock; the idle resource of an enterprise’, while Verma (2002) defines inventory as ‘a detailed list of movable goods, such as raw materials, materials in process, finished products, general supplies, equipments, etc., and gives the quantity and value of each item’. Thus, the proper management and control of inventory is an important factor in controlling any firms incurred costs as well as controlling production schedules.
Inventory and the practice of keeping records have long been in existence, hence the difficulty in tracing their history. However, journals that date as far back as 1200 BC have references to inventory/bookkeeping. According to Edward (1960), ‘record keeping can be defined as the practice of inscribing and preserving documents as evidence to support a particular business transaction or the major transactions of a continuing business enterprise’. Though this slightly differs from inventory, the documentation of transactions is the key word here that ties the two terms together, because these documents would contain the quantity and the price of each item being bought or sold, hence monitoring the inventory level of the enterprise, (Güder, et al., 1995).

The origin of the Economic Order Quantity (EOQ) model is traced back to the work of Ford Whitman Harris, an inventor, engineer, author and patent attorney, whose formal education was not beyond high school. This was in an article that was published in 1913 in a magazine called ‘Factory: The Magazine of Management’. Harris’ original EOQ paper was lost from sight for many years until its rediscovery in 1988 (Erlenkotter, 1989). Some journals and documents however attribute the discovery of the EOQ formula to Wilson in 1918(Panda et al, 2005).

According to Erlenkotter (1990) the EOQ model is so well known that we accept its basic structure as obvious. In 1913, however, it was a modelling achievement of classical elegance. The simple square-root formula for the optimal order quantity followed directly from Harris’s assumption of a continuous constant rate for demand and his recognition of the need to balance tangible inventory costs against the tangible costs for ordering. Versions of Harris’ depiction of cost tradeoffs in the “manufacturing quantities curves” have appeared in all standard works on the subject. Even his original presentation of the model, which defines costs on a per item basis, is easier to interpret than the standard textbook development, which expresses costs as an average per unit of time.

2.0 Model Development

2.1 Theoretical Framework
The managers in charge of purchasing are saddled with the task of answering the questions that demand on inventory control pose. Questions such as determining the length of production period, how much to stock up to keep production running, how many times an order should be placed, what conditions would be required to keep the inventory without rapidly deteriorating and the associated cost of storage conditions, are examples of questions that managers try to find the solutions.

Before production begins, a forecast is calculated from previous sales records. This forecasted demand is translated to the production figure/target time period. The next step is to determine the size of raw materials that would be required to achieve this target. Usually an over-shot of these are ordered for to cater for losses due to waste, spoilage, deterioration, set-backs in the delivery time. All of these raw materials have ordering/holding costs as well as storage conditions which are associated holding costs. Based on the parameters mentioned above, i.e. demand, ordering costs and holding costs, the Economic Order Quantity (EOQ) helps to determine the optimal order quantity that yields the minimum total cost of all the purchasing alternatives.

The problem being considered here is a multi-item inventory problem with a budget constraint. For each project, two cases are considered:

Case 1: The calculation of the EOQ model with any constraints

Case 2: The calculation of the EOQ model with the Company wishes to reduce its expenditure on purchase by 20%, thus the budget constraint.

2.2 Model Assumptions
The following assumptions have been made regarding the mathematical model:

- Lead time is not taken into consideration and assumed that the purchased stock will be available for use when required.
- All purchased stock is efficiently used up with no wastage or losses.
- Ordering cost does not vary with quantity.
- Price of materials is fixed over the study period and there are no quantity discounts.
- Demand is deterministic, constant and is known.
- All goods produced are sold within the time period given.
2.3 Formulation of the Economic Order Quantity Model

Notations

Let:

- \( D_i \): The \( i \)th raw material
- \( C_{oi} \): Demand for the \( i \)th raw material
- \( C_{hi} \): Ordering cost for the \( i \)th raw material
- \( C_{ui} \): Inventory carrying costs associated with the \( i \)th raw material
- \( p_i \): Price per unit of the \( i \)th raw material
- \( B \): Cash available for the purchase of raw materials
- \( Q_i \): Order quantity for the \( i \)th raw material
- \( Q_{oi} \): Economic Order Quantity for the \( i \)th raw material

2.4 Objectives of the Model

This project considers two decision situations:
- A situation where the optimal order quantities of the raw materials are desired and there is no constraint.
- A situation where a budget constraint exists in the determination of the optimal order quantity.

3.0 Objective Function Development

3.1 Case 1 (No Constraints)

Ordering costs for the \( i \)th item = No. of Orders \times Ordering costs/order

\[ \text{annual demand} = \frac{\text{order quantity}}{\text{ordering quantity}} \times \text{ordering costs/order} \]  

Therefore, the annual total inventory cost is given by:

\[ \sum_{i=1}^{I} \frac{D_i}{Q_i} \times C_{oi} \]  

Annual inventory holding costs for the \( i \)th item = Average inventory \times inventory carrying cost

\[ = \frac{Q_i}{2} \times C_{ui} \]

where \( Q_i = \text{(Max Inventory - Min Inventory)} \), and \( C_{ui} \) is the inventory carrying cost.

3.2 Case 2 (Budget Constraint)

In the scenario where a budgetary constraint exists, and it is desired to minimize total cost, the objective function becomes

\[ \text{Minimize } T_c = \sum_{i=1}^{I} \left( \frac{D_i C_{oi} + Q_i C_{hi}}{Q_i} \right) \]

Subject to the associated budget constraint.
\sum_{i=1}^{I} P_i Q_i \leq B \quad \ldots 13

where \( Q_i > 0 \), for all \( i = 1, 2, \ldots, I \).
Thus the Lagrangian function is formulated as follows:

\[ L(\lambda, Q_1, Q_2, \ldots, Q_I) = \sum_{i=1}^{I} \left( \frac{D_i C_{gi}}{Q_i} + \frac{Q_i C_{hi}}{2} \right) - \lambda \left( \sum_{i=1}^{I} P_i Q_i - B \right) \quad \ldots 14 \]

where \( \lambda (\geq 0) \) is the Lagrange multiplier, (Kim and Kim, 2000).
The optimal values of \( Q \) are determined by finding the first partial derivatives of equation (14) with respect to \( Q_i \) and \( \lambda \), then solving for \( Q_i \):

\[ \frac{\partial L}{\partial Q_i} = \sum_{i=1}^{I} \left( \frac{C_{gi} D_i}{Q_i^2} + \frac{Q_i C_{hi}}{2} \right) - \lambda \sum_{i=1}^{I} P_i = 0 \quad \ldots 15 \]

\[ \frac{\partial L}{\partial \lambda} = - \sum_{i=1}^{I} P_i Q_i + B = 0 \quad \ldots 16 \]

Solving equations (15) and (16) simultaneously give

\[ Q_i^* = \frac{\sqrt{2D_i C_{gi}}}{\sqrt{C_{hi} - 2\lambda^* P_i}} \quad \ldots 17 \]

\[ \sum_{i=1}^{I} P_i Q_i - B = 0 \quad \ldots 18 \]

Solution (18) serves as a check as to whether or not the equation has converged or not. From the assumptions made above, the graphical representation of the EOQ model is shown in Figure 1.

4.0 Model Description and Validation

The company under study is a leading manufacturer of household items such as detergents, baby products, etc. as well as candies. The company's department studied for the purpose of this project is the procurement section of their candy-production. Their candy product range includes fresh, vanilla and lemon sweets. Each of these sweet varieties requires eleven raw materials that are sourced both locally and internationally. Seven of the raw materials are common to all the three sweets namely Fat, Citric acid, Natural Oil, Sucrose, Sugar, Talcum and Cocoa butter and only one raw material is common to two of the sweet brands. The raw material is Ascorbic Acid while the brands are Lemon and Vanilla (see Table 1). Each of these sweet varieties has a recipe which states how much of any particular raw material of these sweets should be used in the mix.

4.1 Data Collection and Analysis

Data was collected from the procurement division of the candy factory. The data collected includes the annual demand, price per unit, ordering costs and holding costs for each of the raw materials and are shown in Table 2.

It is desired by the decision makers to find what quantities of the 18 raw materials to order such that the total cost of inventory is minimized.

![Figure 1: Basic Inventory Model](image)
Table 1: Raw materials required for each of the sweets.

<table>
<thead>
<tr>
<th>Fresh</th>
<th>Vanilla</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fat</td>
<td>Fat</td>
<td>Fat</td>
</tr>
<tr>
<td>Citric Acid</td>
<td>Citric Acid</td>
<td>Citric Acid</td>
</tr>
<tr>
<td>Natural Oil</td>
<td>Natural Oil</td>
<td>Natural Oil</td>
</tr>
<tr>
<td>Sucrose</td>
<td>Sucrose</td>
<td>Sucrose</td>
</tr>
<tr>
<td>Sugar</td>
<td>Sugar</td>
<td>Sugar</td>
</tr>
<tr>
<td>Talcum</td>
<td>Talcum</td>
<td>Talcum</td>
</tr>
<tr>
<td>Cocoa Butter</td>
<td>Cocoa Butter</td>
<td>Cocoa Butter</td>
</tr>
<tr>
<td>Aloe A</td>
<td>Ascorbic Acid</td>
<td>Ascorbic Acid</td>
</tr>
<tr>
<td>Additive F</td>
<td>Additive V</td>
<td>Flavour L</td>
</tr>
<tr>
<td>Peppermint Oil</td>
<td>Additive Y</td>
<td>Flavour B</td>
</tr>
<tr>
<td>Additive X</td>
<td>Additive F</td>
<td>Sodium Hydroxide</td>
</tr>
</tbody>
</table>

\[
Q_i^* = \sqrt{\frac{2D_iC_{Wi}}{C_{Wi}}} \quad \forall i = 1, 2, ..., 18
\]

Since \( D_i, C_{Wi} \) and \( C_{Wi} \) are known for each of the raw materials, then \( Q_i^* \) can be calculated for each of them. Microsoft Excel® 2007 was used to solve for \( Q_i^* \) and the results are shown in Table 3. The Annual total inventory cost from the adoption of the EOQ model for each raw material is calculated from equation (5) in section 3,

\[
T_{oi} = \frac{D_iC_{Wi} + OC_{Wi}}{Q_i^*} \quad \forall i = 1, 2, ..., 18
\]

The total annual inventory costs for all 18 raw materials is calculated from equation (6)

\[
T_t = \sum_{i=1}^{18} \left( \frac{D_iC_{Wi} + OC_{Wi}}{Q_i^*} \right)
\]

This gives a total value of \( \text{N}13,830,856.78 \).

ii. Case 2: (A scenario where there is a budget constraint)

Upon the adoption of the EOQ model, the organisation still wishes to cut down its expenses to allow for capital to be used on other projects and also due to scarcity of available funds. Thus, management has decided that it wants to reduce its expenditure per purchase by 20%. Therefore, it can

Table 2: List of all raw materials required by the Company

<table>
<thead>
<tr>
<th>Raw Material</th>
<th>Total Volume required (D_i) (kg)</th>
<th>Ordering Cost (C_{oi}) (N)</th>
<th>Holding cost (C_{Wi}) (N)</th>
<th>Price per Unit (p_i) (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fat</td>
<td>780238.15</td>
<td>130,931.54</td>
<td>56.20</td>
<td>14.88</td>
</tr>
<tr>
<td>Citric acid</td>
<td>842454.87</td>
<td>9,286.51</td>
<td>93.50</td>
<td>0.98</td>
</tr>
<tr>
<td>Natural Oil</td>
<td>324102.63</td>
<td>20,292.24</td>
<td>51.50</td>
<td>25.00</td>
</tr>
<tr>
<td>Sucrose</td>
<td>1083915.00</td>
<td>13,786.34</td>
<td>84.90</td>
<td>1.13</td>
</tr>
<tr>
<td>Sugar</td>
<td>746697.00</td>
<td>6,521.45</td>
<td>53.60</td>
<td>0.77</td>
</tr>
<tr>
<td>Talcum</td>
<td>4817.40</td>
<td>141.47</td>
<td>33.80</td>
<td>2.60</td>
</tr>
<tr>
<td>Cocoa butter</td>
<td>513943.54</td>
<td>30,591.72</td>
<td>78.00</td>
<td>5.26</td>
</tr>
<tr>
<td>Ascorbic Acid</td>
<td>390119.07</td>
<td>45,534.96</td>
<td>94.50</td>
<td>11.45</td>
</tr>
<tr>
<td>Additive A</td>
<td>252.91</td>
<td>7.72</td>
<td>74.40</td>
<td>1.35</td>
</tr>
<tr>
<td>Additive F</td>
<td>192696.00</td>
<td>6,960.57</td>
<td>12.80</td>
<td>1.60</td>
</tr>
<tr>
<td>Peppermint Oil</td>
<td>114822.73</td>
<td>31,078.02</td>
<td>95.80</td>
<td>12.00</td>
</tr>
<tr>
<td>Additive X</td>
<td>1770.39</td>
<td>918.42</td>
<td>90.80</td>
<td>23.00</td>
</tr>
<tr>
<td>Additive V</td>
<td>64661.55</td>
<td>49,048.66</td>
<td>90.30</td>
<td>24.81</td>
</tr>
<tr>
<td>Additive Y</td>
<td>1003.63</td>
<td>2.77</td>
<td>77.50</td>
<td>0.09</td>
</tr>
<tr>
<td>Additive F</td>
<td>192696.00</td>
<td>1,342.59</td>
<td>83.80</td>
<td>260.4</td>
</tr>
<tr>
<td>Flavour L</td>
<td>2408.70</td>
<td>1,580.05</td>
<td>29.40</td>
<td>37.29</td>
</tr>
<tr>
<td>Flavour B</td>
<td>88720.45</td>
<td>23,221.28</td>
<td>77.10</td>
<td>14.88</td>
</tr>
<tr>
<td>Sodium Hydroxide</td>
<td>40546.45</td>
<td>2,918.42</td>
<td>73.40</td>
<td>4.09</td>
</tr>
</tbody>
</table>
Table 3: Table showing the EOQ of the raw materials and the associated inventory costs

<table>
<thead>
<tr>
<th>Raw Material</th>
<th>Total Volume (kg)</th>
<th>Total Volume required (kg)</th>
<th>Ordering Cost ($\text{N}^\text{4}$)</th>
<th>Holding cost ($\text{N}^\text{4}$)</th>
<th>EOQ (kg)</th>
<th>Annual Inventory Cost $C_e$ ($\text{N}^\text{4}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fat</td>
<td>390119.07</td>
<td>760238.15</td>
<td>130931.54</td>
<td>56.20</td>
<td>60295.18</td>
<td>3388588.90</td>
</tr>
<tr>
<td>Citric acid</td>
<td>421227.43</td>
<td>842545.87</td>
<td>9285.51</td>
<td>93.50</td>
<td>12936.26</td>
<td>1209540.43</td>
</tr>
<tr>
<td>Natural Oil</td>
<td>162051.31</td>
<td>324102.63</td>
<td>20292.24</td>
<td>51.50</td>
<td>15981.50</td>
<td>823047.47</td>
</tr>
<tr>
<td>Sucrose</td>
<td>541957.50</td>
<td>1083915.00</td>
<td>13786.34</td>
<td>84.90</td>
<td>18762.14</td>
<td>1592908.94</td>
</tr>
<tr>
<td>Sugar</td>
<td>373348.50</td>
<td>746697.00</td>
<td>6521.45</td>
<td>53.60</td>
<td>13479.60</td>
<td>722506.37</td>
</tr>
<tr>
<td>Talcum</td>
<td>2408.70</td>
<td>4817.40</td>
<td>141.47</td>
<td>33.80</td>
<td>200.81</td>
<td>6787.53</td>
</tr>
<tr>
<td>Cocoa butter</td>
<td>257971.77</td>
<td>515943.54</td>
<td>30591.72</td>
<td>78.00</td>
<td>20117.35</td>
<td>1569153.16</td>
</tr>
<tr>
<td>Ascorbic Acid</td>
<td>195059.54</td>
<td>390119.07</td>
<td>45534.96</td>
<td>94.50</td>
<td>19389.66</td>
<td>1832322.75</td>
</tr>
<tr>
<td>Additive A</td>
<td>126.46</td>
<td>252.91</td>
<td>7.72</td>
<td>74.40</td>
<td>7.24</td>
<td>539.01</td>
</tr>
<tr>
<td>Additive F</td>
<td>96348.00</td>
<td>192696.00</td>
<td>6960.57</td>
<td>12.80</td>
<td>14476.67</td>
<td>185501.41</td>
</tr>
<tr>
<td>Peppermint Oil</td>
<td>57411.36</td>
<td>114822.73</td>
<td>31078.02</td>
<td>95.80</td>
<td>8631.23</td>
<td>826872.13</td>
</tr>
<tr>
<td>Additive X</td>
<td>885.20</td>
<td>1770.39</td>
<td>918.42</td>
<td>90.80</td>
<td>189.25</td>
<td>17183.58</td>
</tr>
<tr>
<td>Additive V</td>
<td>32330.78</td>
<td>64661.55</td>
<td>49040.66</td>
<td>90.30</td>
<td>8381.23</td>
<td>756825.07</td>
</tr>
<tr>
<td>Additive Y</td>
<td>501.81</td>
<td>1003.63</td>
<td>2.77</td>
<td>77.50</td>
<td>8.47</td>
<td>636.43</td>
</tr>
<tr>
<td>Additive F</td>
<td>96348.00</td>
<td>192696.00</td>
<td>1342.59</td>
<td>83.80</td>
<td>2484.85</td>
<td>208230.84</td>
</tr>
<tr>
<td>Flavour L</td>
<td>1204.35</td>
<td>2408.70</td>
<td>1580.05</td>
<td>29.40</td>
<td>508.82</td>
<td>14059.44</td>
</tr>
<tr>
<td>Flavour B</td>
<td>44360.23</td>
<td>88729.45</td>
<td>23221.28</td>
<td>77.10</td>
<td>7310.43</td>
<td>56333.93</td>
</tr>
<tr>
<td>Sodium Hydroxide</td>
<td>20273.23</td>
<td>40546.45</td>
<td>2918.42</td>
<td>73.40</td>
<td>1795.63</td>
<td>131799.37</td>
</tr>
</tbody>
</table>

Therefore since it is desired to minimize total cost, the objective statement can thus be stated from equations (21) and (22) as:

\[
\frac{\partial L}{\partial Q_i} = -\frac{C_{oi} D_i}{Q_i^2} + \frac{Q_i}{2} - \lambda p_i = 0
\]

The second equation shows that the budget constraint must be satisfied in equation form at the optimum.

From equation (22) above

\[
Q_i^* = \frac{2C_{oi} D_i}{C_{hi} - 2\lambda^* p_i}
\]

Thus the Lagrangian function is formulated as

\[
L(\lambda, Q_1, Q_2, \ldots, Q_{18}) = \sum_{i=1}^{18} \left( \frac{D_i C_{oi}}{Q_i} + \frac{Q_i C_{hi}}{2} \right)
\]

...19

\[
\lambda \left( \sum_{i=1}^{18} p_i Q_i - N2232425.17 \right)
\]

...20

...21

\[
\frac{\partial L}{\partial \lambda} = -\sum_{i=1}^{18} p_i Q_i + N2232425.17 = 0
\]

...22

...23
Since the equation has two unknowns, the value of $Q^*$ given the constraints has to be found iteratively. The initial value of $\lambda$ is set to 0; decrements will be by a factor of 0.1. Thus for each decrement, $Q^*$ is calculated. The value is then checked for consistency with the constraint; if $\mu Q_v - B$ is not a negative number, that is, it is still greater than $B$, the iteration continues. The iteration is done 6 times with Microsoft Excel™ 2007, and it was seen that the optimal value was within the range of $0.4 < \lambda < 0.5$. The values are then adjusted until the value closest to zero without passing it is achieved. This value for $\lambda$ is 0.4935. The final iteration is shown in Table 4.

The total cost obtained from this solution is N2,232,386.00 which is less than B. Thus the constraint can be said to be satisfied.

6.0 Discussion

The EOQ model developed was used to determine the optimal order quantity for raw materials in the candy division of a factory. The application of the model shows that it is possible to reduce total inventory costs incurred by the factory. The resulting costs were compared against the existing procurement schedule of the factory. The case study company currently purchase all the required raw materials every 2 months and store, which results in an annual inventory costs of N35,097,340.83. The adoption of the EOQ model shows that this value can be reduced to N13,850,856.78, a decrease of N21,246,484.05. This translates to a 60.54% decrease in inventory costs.

In the case where there is a budget constraint, a Lagrangian multiplier was introduced to find the optimal order quantity that minimized total cost, but kept the total procurement costs within the budget limits. The equation was solved iteratively and yielded a total annual cost of N2,232,386.00.

7.0 Conclusion and Recommendations

The development and application of the EOQ model shows that it is not sufficient to arbitrarily decide
Determination of the Optimal Order Quantity for Multi-Item Inventory

when and how much to purchase, but systematic calculation must be done to know the order quantities and thus the frequency of ordering.

The model presented is a generic model, meaning it could be applicable to any other manufacturing company. It is therefore imperative that factories adopt the use of the EOQ model in finding the optimal order quantities for their required raw materials to increase their profitability.

References


Telsch, R., Martindale 1998, Industrial Engineering and Production Management (First Edition), S. Chand & Company Ltd. India
